Influence of individual variations of particle image intensities on iterative image deformation in PIV

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ABSTRACT

Individual variations of intensity of tracer particles, e.g. due to out-of-plane displacements between exposures, strongly limit the achievable accuracy of correlation based PIV processing. The RMS error originated by this effect correlates with the spatial resolution that can be achieved with the processing algorithm making especially high-resolution algorithms like iterative image deformation affected by this error.

1. INTRODUCTION

Particle image velocimetry (PIV) has become the prime choice for processing image-based flow measurements in fluid dynamics experiments. The basic algorithm of digital PIV processing utilizes the cross-correlation of image sub-spaces (interrogation areas) for local displacement estimation from two consecutively acquired images of a tracer-particle-laden flow.

A variety of image processing techniques has been developed in the past to significantly improve both, the accuracy of the particle displacement measurement beyond the nominal resolution of the optical sensor and the spatial resolution beyond the nominal averaging size of image sub-spaces to be correlated. These include the image deformation techniques [1, 2, 4, 7, 11, 12, 13, 15, 16, 17], where the entire images are deformed accordingly to the assumed velocity field before the sub-division into interrogation areas. With a high overlap of neighboring interrogation areas in combination with an iterative correction of residual displacements of the estimated velocity the spatial resolution is governed by the grid spacing without losing the robustness of the large interrogation areas. Therefore, this method is gained to improve the achievable spatial resolution of the PIV processing. Instabilities of this technique, occurring for high overlaps of interrogation areas can be avoided either by applying appropriate spatial filters to the estimated velocity field or the application of appropriate windowing functions to the interrogation areas, which have frequency responses with only positive values.

At the same time, with iterative window shift and deformation or image deformation techniques, an excellent accuracy of the displacement estimation of the order of 0.01 pixel or better has been reported [3, 5, 10]. These results have been achieved based on synthetic test images. In contrast, the application to real images from experiments shows less optimistic results, where the limit usually observed is about 0.1 pixel. Only under special conditions, like in two-dimensional flows with carefully aligned light sheets, can better accuracy be achieved [6].

One reason for the different achievable accuracies in simulations and experiments is the fact that in experiments, particles usually change their position within the light sheet (Fig. 1a). Therefore, the particles are illuminated differently in the two consecutive exposures. Additionally, the different illumination is individually different for each particle due to

Figure 1: Particles moving through a light sheet with an intensity profile: in a) the particles have an out-of-plane velocity component and in b) there is a two-dimensional flow aligned with the light sheet plane (only in-plane velocity components).

Figure 2: Examples demonstrating individual particle intensity variations (marked regions, detail of public PIV images from the PIV challenge 2003, case A, axisymmetric turbulent jet in stagnant surrounding, images A001a and A001b).

Figure 3: Two images (I and II), each consisting of two particle images (Airy discs) with varying relative intensity of overlapping particle images, are cross-correlated (CC with lines of zero displacement in x and in y direction respectively and with the correlation maximum marked with a black dot). The particles are at identical positions in the two images (no displacement between the images). The correlation peak has a shifted maximum location. a) Gaussian light sheet profile and b) top-hat profile.
their different starting positions perpendicular to the light sheet plane. The result is an individual variation of particle intensities even in a homogeneous flow without any velocity gradient. Intensity variations can easily be seen in images from a variety of PIV applications, where some particles become brighter between the two exposures, whereas other particles, even if close by, become darker (Fig. 2). Simulations often assume that different particles can have different intensities, but not that the intensities can vary between subsequent exposures, which can be realized in experiments only in two-dimensional flows with light sheets exactly aligned parallel to the flow field (Fig. 1b).

In [8] and [9] the influence of individual intensity variations has been studied for classical FFT methods including windowing, direct correlation methods and iterative methods with sub-pixel shift and deformation of the interrogation windows. In this study, after a short introduction of the effect of the intensity variations, the strong influence especially on high-resolution PIV methods such as iterative image deformation is investigated. Both aspects will be shown, the gain of resolution by iterative image deformation and the loss of accuracy due to individual variations of particle intensities.

2. EFFECT OF VARYING INTENSITIES

In PIV, the displacement of particle patterns between consecutive images is obtained from the peak position in the two-dimensional cross-correlation plane of the two images. Assuming a certain number of imaged particles in the interrogation area, each with different intensity, but with the same relative intensity in the two consecutive images and no truncation at the edges of the interrogation areas, the correlation peak is at the correct position, even if the particle images overlap and if the intensity of one entire image is scaled by a constant factor.

This holds true also for the correlation of images with different relative amplitudes of the particle images, as long as the particle images do not overlap. With overlapping particle images and varying relative amplitudes (Fig. 3a), the maximum position of the correlation peak is shifted, yielding a biased displacement estimate, depending on the amplitudes of the particle images, widths, and overlap. The effect of an shifted correlation peak position can be seen also for a top-hat light sheet profile, if one of the overlapping particle images is present in only one of the two exposures (Fig. 3b).

The consequence for PIV image processing is an additional error in displacement estimates, if the intensities of particle images vary between the consecutive PIV images, while the particle images overlap. This error is especially large for de-focussed particle images (where the particle images tend to overlap), in the case of misaligned light sheets or flows with out-of-plane motion of the particles (where the illumination of individual particles changes between the two light pulses) and, for high-resolution PIV analysis, as will be shown in the next section.

To demonstrate the dominating influence of the intensity variation on the accuracy of correlation-based PIV algorithms, an iterative window shift method with bi-cubic spline interpolation, widely accepted as one of the best methods so far, is used exemplarily in a computer simulation for a top-hat light sheet profile and random (equally distributed) particle intensities. In Fig. 4 the total RMS error over the particle image diameter is shown for three test cases: (i) For only in-plane motion (without noise) the RMS deviation clearly drops below 0.01 pixel for particle image diameters larger than 3 pixels. For smaller particle images the effect of under-sampling occurs and limits the achievable estimation accuracy to about 0.1 pixel for a particle image diameter of 1 pixel. (ii) For only in-plane motion, but with photon noise (1000 photo electrons giving about 32 electrons noise, 10 electrons per count then give an corresponding image intensity of 100) and quantization noise (only integer counts), the RMS error in the range above 3 pixels is significantly larger than in the previous case, but still below 0.01 pixel. The range below 3 pixel particle image diameters is still dominated by under-sampling. (iii) An out-of-plane component of the displacement (here 1/4 of the light sheet thickness) has a much stronger influence than the noise. The out-of-plane motion, even without noise, limits the achievable accuracy to about 0.04 pixel when the particle image diameter is at its optimum of about 2 pixels.

In the following section the strong influence of the intensity variation especially for high-resolution PIV methods will be demonstrated.

3. RESULTS

3.1 Simulation

Before the influence of the particle image variations will be investigated, a proof of the gain of resolution by image deformation is shown. Therefore, a series of 100 pairs of PIV images with 512×512 pixels each has been generated with a random displacement field (two-dimensional, pixel-resolution, no out-of-plane displacement). The particle images have random (equally distributed) intensities. Airy disk profiles with 3 pixels diameter (first zero value). The images have been analyzed with an iterative window shift and first-order deformation [14] with 32×32 and 16×16 pixels window size and an iterative image deformation with a triangular weighting of each PIV window (exactly as in [11]) of 32×32 pixels size. To isolate the effect of decreasing the effective window size by weighting, the triangular weighting function has also been applied to the iterative window shift and deformation with a 32×32 pixels window. All methods use an 8×8 velocity estimation grid.

From the individual displacement estimates, which are interpolated with bi-cubic splines and re-sampled at all pixel positions, and the simulated displacement, which originally is given for all pixel positions, a two-dimensional coherent
frequency response

\[ C_{ij} = \frac{\langle U_{\text{est},ij}^* U_{\text{sim},ij} + V_{\text{est},ij}^* V_{\text{sim},ij} \rangle}{\langle U_{\text{sim},ij}^* U_{\text{sim},ij} + V_{\text{sim},ij}^* V_{\text{sim},ij} \rangle} \]  

is calculated, where \( U_{\text{sim},ij} \) and \( V_{\text{sim},ij} \) are the two-dimensional Fourier transforms of the simulated \( u \) and the \( v \) displacement fields, \( U_{\text{est},ij} \) and \( V_{\text{est},ij} \) are the estimated counterparts, the \( \ast \) denotes the conjugate complex and \( \langle \rangle \) denotes the ensemble average. The products and the coherence frequency function are calculated element-wise for the two-dimensional functions. From the two-dimensional coherent frequency response function a common (one-dimensional) one is derived by iteratively optimizing a one-dimensional function \( c_f \) so that the component-wise products \( c_f c_f \) fit best the two-dimensional function \( C_{ij} \) with minimum L_2 norm.

Fig. 5 shows the results for the four investigated estimation procedures. With a rectangular weighting window, the frequency response clearly drops below zero at 1/32 pixel or 1/16 pixel corresponding to the interrogation area size of 32 x 32 or 16 x 16 respectively. The triangular weighting window applied to a 32 x 32 interrogation window leads to a frequency response function with only positive values, while the resolution increases beyond the nominal resolution of the interrogation window size, reaching approximately an effective window size of half the nominal window size. For all three PIV algorithms, the final resolution is defined by the interrogation window size, which does not improve with further iterations. Only the image deformation technique can further improve the spatial resolution, which then is limited by the velocity grid of 8 x 8 pixels.

In the second simulation a series of 100 PIV image pairs with a homogeneous displacement field with random displacement in the range of \( \pm 1 \) pixel in both in-plane directions and with a varied out-of-plane component has been generated. The simulated light sheet has a top-hat profile. The particle images have random (equally distributed) intensities, Airy disk profiles with 3 pixels diameter (first zero value). The images have been analyzed with the four procedures as above. From the estimated displacement fields, arranged in an 8 x 8 grid, the total RMS errors have been calculated. To keep the investigations simple and to isolate the influence of intensity variations window deformation has not been implemented here to avoid other effects superimposing.

In Fig. 6 the total RMS error of the four investigated estimation procedures is shown. It increases approximately exponentially with the out-of-plane displacement up to an out-of-plane displacement of about 1/2 of the light sheet thickness. Beyond this value the estimators become unreliable and outliers dominate the RMS error. The ranking of the four procedures directly corresponds to the achievable resolution, yielding larger RMS errors for higher resolution procedures. Obviously, a smaller resolution corresponds to an average from a wider area, which also averages local deviations of the estimates.

### 3.2 Experiment

Verification of the results requires an accurate synchronization of the in-plane and the out-of-plane translation through the light sheet. For best possible flexibility, intensity profiles have been projected into the measurement volume using a video projector and an additional collimation lens (Fig. 7). To achieve stable illumination, LCD technology is preferred. The projector with DLP technology used here realizes individual gray values by pulse width modulation, which causes illumination problems with PIV cameras at short exposure (integration) times. In the present study the exposure time has been set to 0.25 s, which corresponds to 30 illumination cycles of the DLP chip, since it works at a frame rate of 120 Hz. This long exposure time requires small velocities, which have been realized by moving a solid glass block on a 3D translation stage. The glass block has a size of 5 cm x 5 cm x 8 cm and includes 54,000 randomly distributed dots in the inner 3 cm x 3 cm x 6 cm volume, corresponding to a particle density of 1 mm⁻³. The glass block moves along one axis of the translation stage, while the plane of illumination is tilted with respect to the axis of motion. During the translation of the glass block with a constant velocity of 0.1 mm s⁻¹ through the observation area of the camera, a series of 80 images with 480 x 480 pixels size is taken at a frame rate of 0.8 Hz. By choosing the number of frames between the two frames to be correlated, different out-of-plane components can be imitated. For better statistics, 9 x 11 displacement vectors have been calculated for each pair of images with non-overlapping interrogation areas. Furthermore, the results for all image pairs with the same number of frames between them, selected from the original series of 80 images, have been averaged.

Fig. 8 shows the experimentally obtained total RMS errors for
the four investigated estimators as a function of the out-of-plane displacement. The order of magnitude and the ranking of the four estimators agree with the above simulation, verifying both the effect of the intensity variations and the simulation procedure. However, there is a quantitative deviation, which possibly originates in a mechanical uncertainty of the traversing stage and a cross-illumination of markers outside, but close to the illumination sheet. Furthermore, the markers in the experiment have all similar intensities, which was not the case in the simulation.

4. CONCLUSIONS

The effect of particle image intensities varying individually between the two consecutive images on the obtainable accuracy of image deformation PIV methods has been investigated. The larger effect of the intensity variations is the prize to be paid for the higher resolution that can be achieved with this type of PIV processing algorithms. Since no mean against the error by intensity variation has been found so far, the achievable resolution and accuracy must be balanced by either minimizing the out-of-plane displacement between exposures or using large enough averaging areas. The later, however, contradicts the purpose of high-resolution PIV methods as e.g. the iterative image deformation.

REFERENCES

[8] H Nobach and E Bodenschutz. Limitations of accuracy in PIV due to individual variations of particle image intensities. accepted by Experiments in Fluids.